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ABSTRACT

Two units make up this material. They focus on applications of second-order difference equations to American politics. The goals of the first module include helping the user understand: 1) the difference between a theoretical construct and an observable; 2) the nature of choices which exist in modeling any particular empirical phenomenon; and 3) more about difference equations. The second module encompasses the goals listed above, as it is a continuation of an analysis of Supreme Court decision-making. Exercises are included, with answers, in each of the units. (MP)

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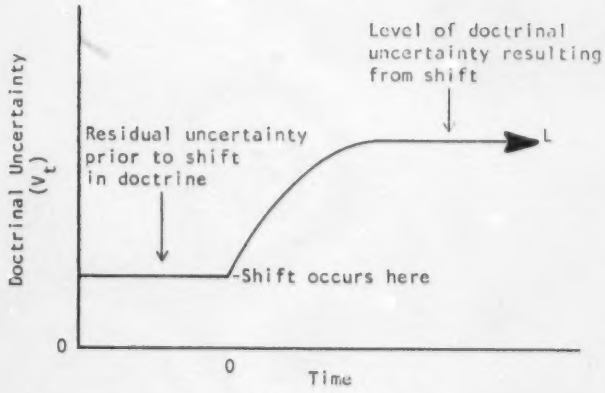
UNIT 306

MODULES AND MONOGRAPHS IN UNDERGRADUATE MATHEMATICS AND ITS APPLICATIONS PROJECT

DISCRETIONARY REVIEW BY THE SUPREME COURT I:

THE MODEL

by Thomas W. Likens



APPLICATIONS OF SECOND ORDER DIFFERENCE EQUATIONS TO AMERICAN POLITICS

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SE 030 231

Intermodular Description Sheet: UMAP Unit 306

Title: DISCRETIONARY REVIEW BY THE SUPREME COURT I: THE MODEL

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Review Stage/Date: III 1/26/79

Classification: APPL SEC ORD DIFF EQ/AMER POL

Suggested Support Materials:

References: See Section 5 of text.

Prerequisite Skills:
For mathematical audiences:
1. A perspective on functions and graphing, including ability to restrict a real valued function to the domain of whole numbers to obtain a sequence.
2. Ability to reason symbolically.

For political science audiences:
1. Previous experience with first-order linear difference equations with constant coefficients.

Output Skills:
1. Understand the difference between a theoretical construct and an observable.
2. Understand the nature of choices which exist in modeling any particular empirical phenomenon.
3. Understand more about difference equations particularly: a) behavior of second order systems, b) use of Δ, E and I as linear operators, and c) solution of linear systems by Cramer's rule.

Other Related Units:
Exponential Models of Legislative Turnover *(Unit 296)*
The Dynamics of Political Mobilization I *(Unit 297)*
The Dynamics of Political Mobilization II *(Unit 298)*
Public Support for Presidents I *(Unit 299)*
Public Support for Presidents II *(Unit 300)*
Laws that Fail I *(Unit 301)*
Laws that Fail II *(Unit 302)*
Diffusion of Innovation in Family Planning *(Unit 303)*
Growth of Partisan Support I *(Unit 304)*
Growth of Partisan Support II *(Unit 305)*
Discretionary Review by the Supreme Court II *(Unit 307)*
What Do We Mean By Policy? *(Unit 310)*

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MODULES AND MONOGRAPHS IN UNDERGRADUATE
MATHEMATICS AND ITS APPLICATIONS PROJECT (UMAP)

The goal of UMAP is to develop, through a community of users and developers, a system of instructional modules in undergraduate mathematics and its applications which may be used to supplement existing courses and from which complete courses may eventually be built.

The Project is guided by a National Steering Committee of mathematicians, scientists and educators. UMAP is funded by a grant from the National Science Foundation to Education Development Center, Inc., a publicly supported, nonprofit corporation engaged in educational research in the U.S. and abroad.

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This unit was presented in preliminary form at the Shambaugh Conference on Mathematics in Political Science Instruction held December, 1977 at the University of Iowa. The Shambaugh Fund was established in memory of Benjamin F. Shambaugh who was the first and for forty years served as the chairman of the Department of Political Science at the University of Iowa. The funds bequeathed in his memory have permitted the department to sponsor a series of lectures and conferences on research and instructional topics. The Project would like to thank participants in the Shambaugh Conference for their reviews and all others who assisted in the production of this unit.

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DISCRETIONARY REVIEW BY THE SUPREME COURT
PART ONE: THE MODEL

1. THE PROBLEM

In 1925, the Congress passed the Judiciary Act which established the discretionary jurisdiction of the United States Supreme Court. The result of this act was the ability of the Court to grant or to deny a hearing by writ of certiorari. It provided the Court with a formal means by which it could control its own policy role (Baum 1977; Frankfurter and Landis 1927; Tanenhaus et al. 1963).

If one examines the types of cases the Supreme Court admits through its discretionary jurisdiction, one often encounters an interesting pattern of change over time. If we take the number of cases the Court hears in the area of reapportionment, for example, and divide by the total number of cases the Court hears that term, we have a measure of the relative frequency with which the Court grants access to that class of cases concerning the issue of reapportionment. If this relative frequency is then plotted across time, a pattern is exhibited in which the frequency of cases starts out at one relatively constant level, then grows rapidly, then levels off briefly, then begins to decay to a new, relatively constant level. The pattern which is observable is idealized in Figure 1 below. Note in addition that there is no particular necessity for the frequency of litigation after the period of growth and then decay to be greater than its initial level. Movement from a higher level to a lower one is certainly possible.

Figures 2 and 3 provide empirical examples in the areas of reapportionment and "search and seizure" cases. The idealized pattern of Figure 1 may be conceptualized as the "central tendencies" of the time-paths in Figures 2 and 3, that is, the smooth trajectory which would result

if the noise (or error) could be removed from the process. It is this idealized process of growth and then decay to a limit which we wish to understand.

More precisely, the question addressed in this module is twofold. First, what explains the pattern of growth and then decay to a limit in the frequency of litigation? And second, how can this process be dynamically modeled?

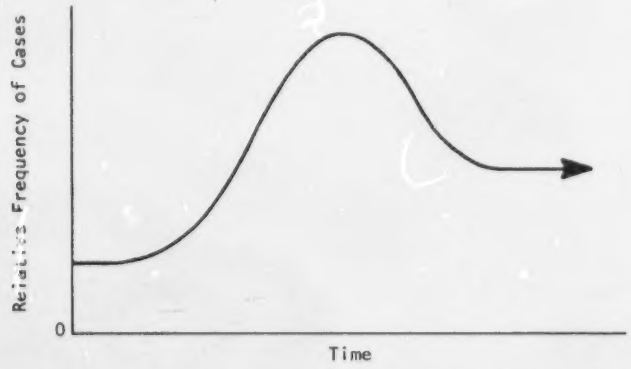


Figure 1. Idealized pattern of change in the relative frequency of cases admitted by the Supreme Court in particular issue areas.

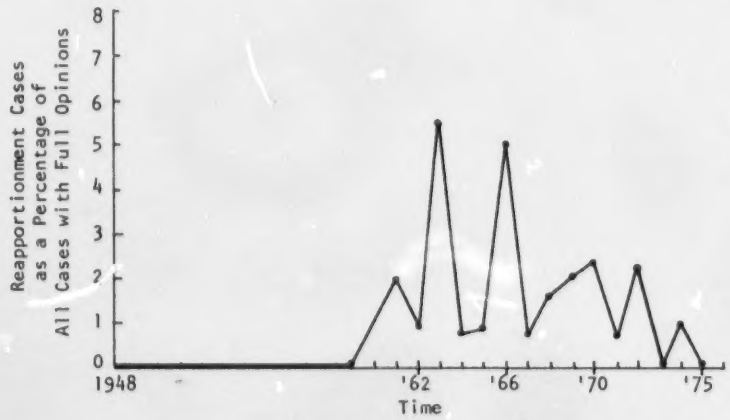


Figure 2. Reapportionment cases as a percentage of all cases with full opinions, 1948-1975 terms. Source: Harvard Law Review, 1949-1976 (November issues).

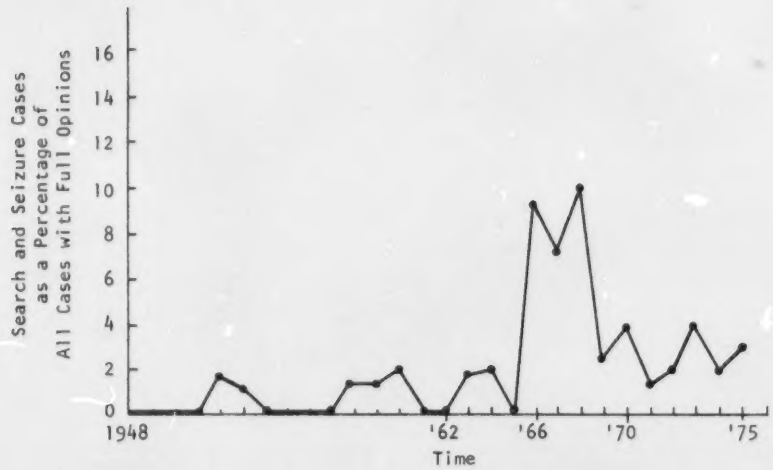


Figure 3. Search and seizure cases as a percentage of all cases with full opinions, 1948-1975 terms. Source: Harvard Law Review, 1949-1976 (November issues).

2. A THEORETICAL EXPLANATION

Suppose the following process occurs. In any particular substantive area of litigation, the Court develops a set of legal norms—doctrines. Because the law is never certain, cases continue to be admitted to the Court in order to interpret the extension of these doctrines to particular sets of facts. Thus, during periods of "doctrinal certainty," there will be an associated level of litigation (a more or less fixed frequency of discretionary review in this area by the Court). Periods of fixed levels of litigation are exhibited in Figure 1 in the flat trajectory before the period of rapid growth and *after* the period of decay.

It is obvious, however, that major "doctrinal shifts" take place. Reapportionment provides a classic example of an extreme doctrinal shift: the Court moved from the argument that it could not hear reapportionment cases at all in *Colgrove* v. *Green* (328 U.S. 1 [1946]) to the position voiced in *Baker* v. *Carr* (369 U.S. 186 [1962]) that reapportionment cases were no longer a political issue and were, therefore, within the Court's jurisdiction.

Notice in Figure 2 that prior to the doctrinal shift in 1962 (*Baker* v. *Carr*), the frequency of reapportionment cases was relatively fixed (at zero). Then with *Baker* v. *Carr*, a major shift in interpretation occurs. As a consequence, (1) the legal public becomes uncertain as to what types of reapportionment issues the Court will hear; and (2) the Court seeks to reestablish certainty in the law by developing a new set of interpretations concerning reapportionment—that is, a legal doctrine.

Because most people are demanding access to the Court, and because the Court itself is attempting to clarify and codify its position, the frequency of litigation temporarily increases. As more cases are tried and a body of precedent is established, however, the law becomes more settled. The temporary increase in litigation thus subsides as the law once again becomes more certain. Doctrinal development, then, will be accompanied by our growth-decay pattern of litigation.

A Dynamic Model of the Process[1]

The preceding argument provides a basic explanation of the growth-decay pattern of discretionary review by the Supreme Court. It is possible to gain a better understanding of the implications of this argument, however, by developing a formal model of the process which has been described. Formalization forces us to articulate the explanation with greater precision. It provides a significantly more powerful linguistic structure. And, perhaps most importantly, formalization makes it possible for us to ascertain the deductive consequences of our intuitions about discretionary review by the Court.

[1]My thanks to Richard Singer, Department of Mathematics, Webster College, for his helpful comments concerning the formalization presented here.

The first part of our argument is that a significant shift in doctrine produces a major increase in the level of uncertainty about the law. This situation would be approximated graphically by Figure 4 below, if no additional litigation occurred after the doctrinal shift.

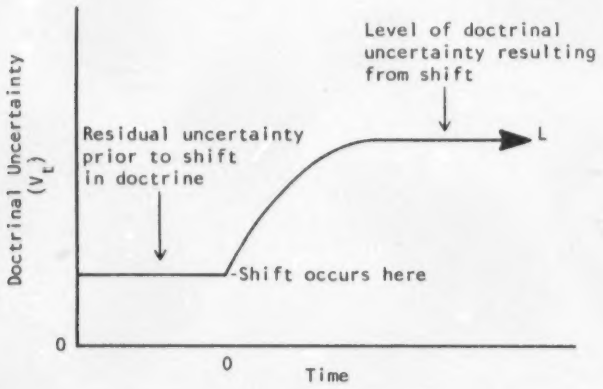


Figure 4. Increase in uncertainty resulting from doctrinal shift.

Notice in Figure 4 that prior to the doctrinal shift there is some residual level of uncertainty in the law. Denote as V_t *the level of uncertainty associated with the doctrinal shift, given the specific assumption that no further litigation takes place.* Then the residual uncertainty at the time of the shift is given by the initial condition V_0. At time $t = 0$ a shift occurs, and in the absence of further litigation, uncertainty (V_t) rapidly grows to some new level, which we will denote here as L. The quantity L may be interpreted as the maximal level of uncertainty which can be produced by any particular doctrinal shift.

We will treat V_t as a pure theoretical construct—an *unobservable* state of the system. Were we to seek an empirical measure of uncertainty in court behavior we would face a difficult or impossible task. And further,

V_t represents uncertainty *given a particular hypothetical assumption*, that no further litigation takes place. Since further litigation usually does ensue, V_t is a concept which does have empirical import but which is not actually present under normal circumstances.

Since Supreme Court terms are discrete events, we will treat the domain of V_t as the non-negative integers. That is, V_t is defined for all values of $t = 1, 2, \ldots, \infty$.

The dynamics of Figure 4 may be formalized by the simple expression

$$(1) \qquad \Delta V_t = g(L - V_t).$$

The state V_t gives the level of uncertainty associated with the law; L gives the upper limit on this uncertainty; and the parameter g is related to the rate at which V_t approaches this upper limit (g is the rate at which the difference between L and V_t is reduced). It is important to note here that g is a constant representing a particular number. Given the situation being modeled, an alternative formalization might treat g as a function of time. We will treat g here as a constant, however, for two reasons. First, we seek an *elementary* explanation (one which is mathematically simple). And second, a standard mathematical technology exists for the analysis of linear difference equations with constant coefficients; no such standard technology exists when we allow the coefficients to be nonconstant terms.

The theoretical structure of the model requires that we specify some range for uncertainty (V_t). Here a convenient assumption is uncertainty is bounded within the zero-unity state-space. In other words, if V_t equals zero then there is no uncertainty present; as V_t approaches unity, the law becomes increasingly ambiguous. At $V_t = 1$, the Court's position would be essentially random—totally unpredictable. It is important to note that this choice

for the range of V_t is purely conventional: unlike our assumption that g is a constant (which makes a specific substantive claim), the restriction of V_t to the zero-one interval is irrelevant to the faithfulness of the model to the substantive phenomenon. Our constraint, then, is given by

(2) $\quad 0 \leq V_t \leq 1.$

If V_t is to have its intended interpretation, we must also impose the constraint that

(3) $\quad 0 < g < 1.$

This guarantees the asymptotically increasing time-path specified in Figure 4.

Thus far we have assumed that nothing reduces the uncertainty which potentially results from a doctrinal shift. Clearly this is false since new litigation ensues (and indeed, it does so at an abnormally high level). The purpose of this litigation, of course, is uncertainty reduction.

Opinions are issued which better articulate, extend and clarify the Court's position. New doctrine, in other words, is developed. Uncertainty is reduced as the law becomes more settled. Figure 5 below illustrates the process.

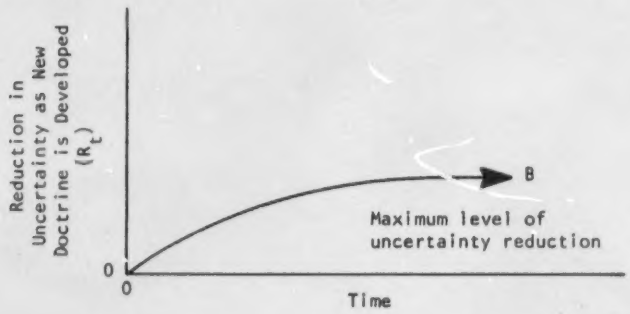


Figure 5. Reduction in uncertainty resulting from doctrinal development.

Note from this illustration that the amount of uncertainty reduction (R_t) has an initial value (when the shift occurs) of $R_0 = 0$. As doctrine is developed, uncertainty is reduced more and more, to some upper limit, B. The curve may be approximated by the dynamic equation

(4) $\quad \Delta R_t = f(B-R_t)$

(5) $\quad R_0 = 0.$

Again, R_t denotes the reduction in uncertainty which has occurred at time t as a consequence of doctrinal development. The parameter B specifies an upper limit (since some uncertainty is always present). And the parameter f gives the rate at which uncertainty reduction occurs as new cases are heard by the Court.

As with the process of uncertainty generation, we will treat this process of uncertainty reduction, denoted here in the dynamics of R_t, as theoretical concept specifying an unobservable state of the system. Similarly, we impose the constraints

(6) $\quad 0 \leq R_t, B \leq 1$ $\quad$ and

(7) $\quad 0 < f < 1.$

Two processes thus occur simultaneously: an increase in uncertainty generated by the shifting interpretation of the Court, and a decrease in uncertainty as new cases are heard and new doctrine is articulated. The *actual* level of uncertainty in the law, then, is given by the difference between these two states of the system. Notice that we have an implicit assumption: even though these processes occur simultaneously, they are independent and can be summed to obtain the actual level of uncertainty. This assumption, of course, is not the only one which is possible. But given our relative ignorance about other formalizations, this assumption is utilized since it keeps the mathematics of the model as tractible as possible.

We will denote this *actual level of uncertainty* as A_t, such that

(8) $A_t = V_t - R_t$.

Since uncertainty is never negative, we logically require that

(9) $V_t > R_t$.

The reader should convince himself/herself that an implication of Inequality (9) is

(10) $L \geq B$.

Question 1: Why is $L \geq B$ an implication of the constraint $V_t \geq R_t$?

Both the Court and the public respond to doctrinal uncertainty. When the law is highly ambiguous the legal public is much more likely to petition the Court for a hearing than when the law is very settled. Similarly, the Court is more likely to grant access to its decision-making powers when there is confusion as to the law or the Court's interpretation of it.

This implies that the observed frequency of litigation in particular substantive areas will be related to the actual level of doctrinal uncertainty which exists at that point in time. We will make the simple assumption that the frequency of litigation, denoted here as X_t, is directly proportional to the level of uncertainty (A_t) which exists at that point in time. We may then write

(11) $X_t = pA_t$.

Thus, X_t denotes *the observed frequency of litigation* and should exhibit the growth-decay pattern which we hope to explain. Notice that X is a function with a discrete domain, the nonnegative integers. Similarly, we take A to be a function whose domain is the nonnegative reals. The parameter p specifies the rate at which uncertainty

produces litigation. In effect, p provides a measure of the sensitivity of the Court: if the Court is highly sensitive to doctrinal uncertainty and the societal demands for access which uncertainty generates, then p will be large. If the Court becomes increasingly unresponsive, p tends toward zero.

The system of dynamic equations which characterizes the basic argument presented above is thus

(12) $\Delta V_t = g(L-V_t)$

(13) $\Delta R_t = f(B-R_t)$

(14) $A_t = V_t - R_t$

(15) $X_t = pA_t$.

It is convenient to substitute (14) into (15), thereby eliminating one equation (and one state). The system thus reduces to

(16) $\Delta V_t = g(L-V_t)$

(17) $\Delta R_t = f(B-R_t)$

(18) $X_t = p(V_t-R_t)$,

subject to the constraints

(19) $V_t \geq R_t$, $t \geq 0$

(20) $L \geq B$, and

(21) $0 < f, g < 1$.

Question 2: What would happen to the time-path of V_t if g were equal to 3?

Qualitative Behavior of the Model

One of the easiest ways to understand how this complicated-looking model behaves is to decompose its

elements graphically. A companion module dissects the model's analytic properties. Here we will conclude by showing that the model does exactly what we require: it generates a time-path for X_t (the actual frequency of litigation) which nicely approximates the idealized central tendency which we wish to explain (Figure 1).

Recall that the actual level of uncertainty to which the Court and society responds is given by the quantity $(V_t - R_t)$. Graphically, this quantity may be observed by plotting the time-paths for V_t and R_t and examining the distance *between the two curves*.

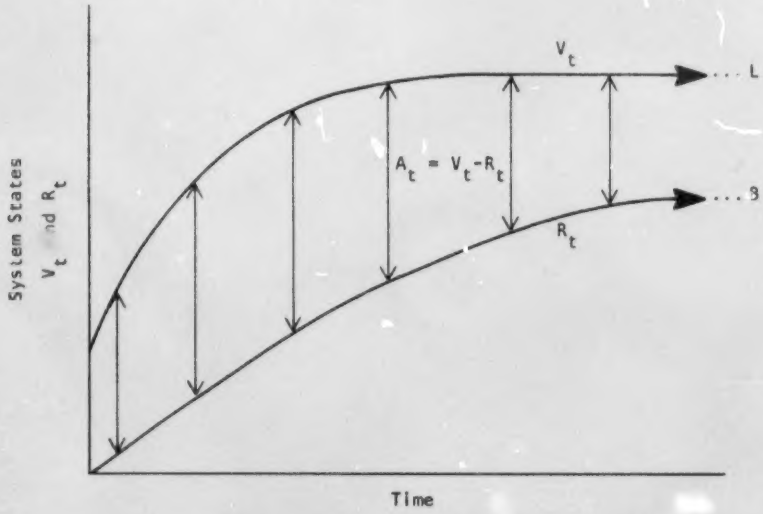


Figure 6. Difference between V_t and R_t gives actual level of uncertainty, A_t.

Notice from Figure 6 that the distance between V_t and R_t begins at the level $(V_0 - R_0)$, then increases, and then begins to decrease. Finally, as V_t and R_t both approach their equilibrium points, L and B, the distance between V_t and R_t is given by the quantity (L-B).

It should be obvious that not all monotonically growing curves for V_t and R_t will produce the growth-decay pattern we require. In Figure 7 below, for example, the constraint $V_t \geq R_t$ holds, but the distance between V_t and R_t does not replicate the growth-decay process (it does just the opposite—it decays and then grows).

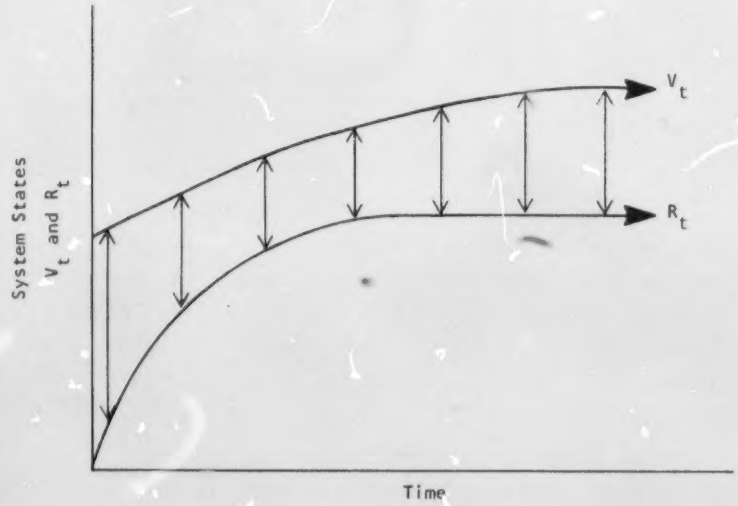


Figure 7. Pattern of litigation generated when constraint $g > f$ is violated.

By comparing Figures 6 and 7, it is obvious that *the only way our characteristic growth-decay pattern will emerge is if V_t grows to its upper limit L at a faster rate than R_t grows to its upper limit B.* In terms of the parameters of the model, then, the implication is that *g must be greater than f to produce the empirically observed central tendency.*

Substantively, this makes perfectly good sense. A doctrinal shift produces uncertainty *very rapidly.* Thus, V_t grows to its upper level very quickly. Reduction in uncertainty, R_t, is an inherently slower process: new

cases must be tried, new opinions must be delivered, a body of precedent established, before a well-articulated interpretation is developed. Hence, the unanticipated consequence, g > f emerges from the formalization.

Question 3: Draw the typical pattern of change which would result if g were less than f and all other constraints hold.

We may conclude by considering one additional feature of the process—the relationship between its starting and endpoints. A well-developed model should be able to produce two distinct qualitative behaviors, illustrated below.

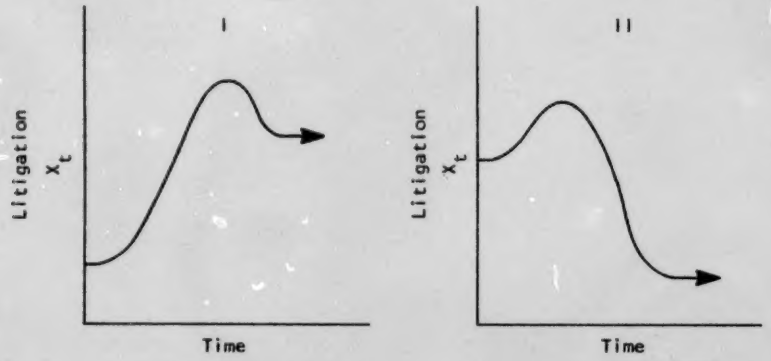


Figure 8. Patterns of doctrinal development when pre- and post-period levels differ.

In the left-hand path, we begin at a *low level of litigation*, move through the process of doctrinal development, and then litigation stabilizes at a *higher level than was typical initially*. In the second example, doctrinal development produces *a level of litigation which is less than was typical initially*.

Returning to Figure 5 it is easy to see how both trajectories are possible with the model. Specifically, we

need merely to compare the initial conditions (V_0 and R_0) with the endpoints (L and B). The initial level of litigation is given by

(22) $X_0 = p(V_0-R_0)$,

but since $R_0 \equiv 0$, we may write

(23) $X_0 = p(V_0)$.

In words, the level of litigation which exists at the time of the doctrinal shift depends on the residual uncertainty surrounding the (pre-shift) doctrine, V_0, and the Court's responsiveness to this uncertainty, p.

Once the new doctrine is fully articulated, the new residual level of uncertainty is given by the quantity (L-B), and the frequency of litigation by

(24) $X_t = p(L-B)$.

The critical question, then, hinges on how much uncertainty exists prior to the period of doctrinal development, V_0, and how much exists once the new doctrine has been fully articulated, (L-B). Pattern I in Figure 8 occurs if

(25) $V_0 < (L-B)$,

pattern II results if

(26) $V_0 > (L-B)$,

and in the special instance

(27) $V_0 = (L-B)$,

the pre- and post-developmental periods experience the same frequency of litigation.

3. CONCLUSION

In this module we have moved from an empirical puzzle—a systematic pattern of change in litigation—through a

verbal explanation to a mathematical formalization. The model does in fact generate the anomalous pattern of change which we sought to explain. Two unanticipated consequences also arose, concerning (1) the relationship between f and g in producing the growth-decay pattern, and (2) the relationship between V_0 and (L-B) in generating two distinct patterns of pre- and post-developmental levels of litigation. In the next module, the analytic properties of the system are further explored.

4. ANSWERS TO QUESTIONS

1. Since $V_t \to L$ in the long run and $R_t \to B$, then L must be greater that B or else $V_t - R_t$ would not hold.

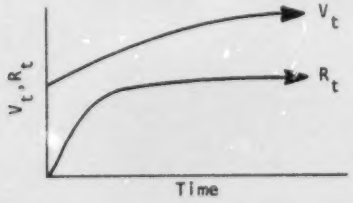
2. If g = 3, the dynamic equation becomes

$$\Delta V_t = 3(L - V_t)$$

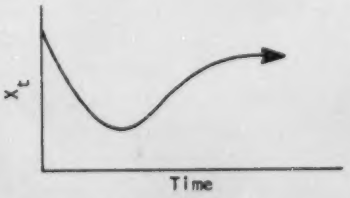
$$V_{t+1} = -2V_t + 3L.$$

V_t would explosively oscillate, thereby violating the construction of the problem.

3. The trajectories would be



which would generate:



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Student: If you have trouble with a specific part of this unit, please fill out this form and take it to your instructor for assistance. The information you give will help the author to revise the unit.

Your Name_____ Unit No._____

Page_____
○ Upper
○ Middle
○ Lower

OR

Section_____
Paragraph_____

OR

Model Exam
Problem No._____
Text
Problem No._____

Description of Difficulty: (Please be specific)

Instructor: Please indicate your resolution of the difficulty in this box.

○ Corrected errors in materials. List corrections here:

○ Gave student better explanation, example, or procedure than in unit. Give brief outline of your addition here:

○ Assisted student in acquiring general learning and problem-solving skills (not using examples from this unit.)

Instructor's Signature_____

Please use reverse if necessary.

STUDENT FORM 2

Unit Questionnaire

Return to:
EDC/UMAP
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Name_____Unit No._____Date_____

Institution_____Course No._____

Check the choice for each question that comes closest to your personal opinion.

1. How useful was the amount of detail in the unit?

____Not enough detail to understand the unit
____Unit would have been clearer with more detail
____Appropriate amount of detail
____Unit was occasionally too detailed, but this was not distracting
____Too much detail; I was often distracted

2. How helpful were the problem answers?

____Sample solutions were too brief; I could not do the intermediate steps
____Sufficient information was given to solve the problems
____Sample solutions were too detailed; I didn't need them

3. Except for fulfilling the prerequisites, how much did you use other sources (for example, instructor, friends, or other books) in order to understand the unit?

____A Lot ____Somewhat ____A Little ____Not at all

4. How long was this unit in comparison to the amount of time you generally spend on a lesson (lecture and homework assignment) in a typical math or science course?

Much Longer	Somewhat Longer	About the Same	Somewhat Shorter	Much Shorter
____	____	____	____	____

5. Were any of the following parts of the unit confusing or distracting? (Check as many as apply.)

____Prerequisites
____Statement of skills and concepts (objectives)
____Paragraph headings
____Examples
____Special Assistance Supplement (if present)
____Other, please explain_____

6. Were any of the following parts of the unit particularly helpful? (Check as many as apply.)

____Prerequisites
____Statement of skills and concepts (objectives)
____Examples
____Problems
____Paragraph headings
____Table of Contents
____Special Assistance Supplement (if present)
____Other, please explain_____

Please describe anything in the unit that you did not particularly like.

Please describe anything that you found particularly helpful. (Please use the back of this sheet if you need more space.)

umap

UNIT 307

MODULES AND MONOGRAPHS IN UNDERGRADUATE MATHEMATICS AND ITS APPLICATIONS PROJECT

DISCRETIONARY REVIEW BY THE SUPREME COURT II:

ANALYSIS OF THE MODEL

by Thomas W. Likens

"... a major shift in interpretation [of the law] signals a period of uncertainty about the Court's behavior, along with increased demand by legal participants for access to the Court. The Court's caseload in this area of litigation thus exhibits a period of growth as the Court attempts to develop a new set of rules--legal norms. Once a new doctrine is articulated, the level of uncertainty about the Court will diminish, and with it the demand for access. Litigation will return to more routine levels."

APPLICATIONS OF SECOND ORDER DIFFERENCE EQUATIONS TO AMERICAN POLITICS

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DISCRETIONARY REVIEW BY THE SUPREME COURT
PART TWO: ANALYSIS OF THE MODEL

by

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Department of Political Science
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Lexington, Kentucky 40506

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Intermodular Description Sheet: UMAP Unit 307

Title: DISCRETIONARY REVIEW BY THE SUPREME COURT II: ANALYSIS OF THE MODEL

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Review Stage/Date: III 1/26/79

Classification: APPL SEC ORD DIFF EQ/AMER POL

Suggested Support Materials:

References: See Section 6 of text.

Prerequisite Skills:
For mathematical audiences:
1. Perspective on functions and graphing, including ability to restrict a real valued function to the domain of the whole numbers to obtain a sequence.
2. Basic understanding of limits of a sequence.
3. Ability to reason symbolically.

For policital science audiences:
1. Previous experience with first order linear difference equations with constant coefficients.

Output Skills:
1. Understand the difference between a theoretical construct and an observable.
2. Understand the nature of nature of choices which exist in modeling any particular empirical problem.
3. Understand more about difference equations, particularly a) behavior of second order systems, b) use of Δ, E and I as linear operators, and c) solution of linear systems by Cramer's rule.

Other Related Units:
Exponential Models of Legislative Turnover *(Unit 296)*
The Dynamics of Political Mobilization I *(Unit 297)*
The Dynamics of Political Mobilization II *(Unit 298)*
Public Support for Presidents I *(Unit 299)*
Public Support for Presidents II *(Unit 300)*
Laws that Fail I *(Unit 301)*
Laws that Fail II *(Unit 302)*
Diffusion of Innovation in Family Planning *(Unit 303)*
Growth of Partisan Support I *(Unit 304)*
Growth of Partisan Support II *(Unit 305)*
Discretionary Review by the Supreme Court I *(Unit 306)*
What Do We Mean By Policy? *(Unit 310)*

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MODULES AND MONOGRAPHS IN UNDERGRADUATE
MATHEMATICS AND ITS APPLICATIONS PROJECT (UMAP)

The goal of UMAP is to develop, through a community of users and developers, a system of instructional modules in undergraduate mathematics and its applications which may be used to supplement existing courses and from which complete courses may eventually be built.

The Project is guided by a National Steering Committee of mathematicians, scientists and educators. UMAP is funded by a grant from the National Science Foundation to Education Development Center, Inc., a publicly supported, nonprofit corporation engaged in educational research in the U.S. and abroad.

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This unit was presented in preliminary form at the Shambaugh Conference on Mathematics in Political Science Instruction held December, 1977 at the University of Iowa. The Shambaugh Fund was established in memory of Benjamin F. Shambaugh who was the first and for forty years served as the chairman of the Department of Political Science at the University of Iowa. The funds bequeathed in his memory have permitted the department to sponsor a series of lectures and conferences on research and instructional topics. The Project would like to thank participants in the Shambaugh Conference for their reviews and all others who assisted in the production of this unit.

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DISCRETIONARY REVIEW BY THE SUPREME COURT
PART TWO: ANALYSIS OF THE MODEL

1. INTRODUCTION

This module is a continuation of an analysis of Supreme Court decision-making. The problem is to explain a particular pattern of growth and then decay to a limit in the frequency of litigation in particular issue areas.

In the previous module, a dynamic model of discretionary review by the Supreme Court was developed. That model is given by the system of dynamic equations

(1) $\Delta V_t = g(L-V_t)$

(2) $\Delta R_t = f(B-R_t)$

(3) $X_t = p(V_t-R_t)$,

subject to the constraints

(4) $0 < V_t, R_t, X_t, g, f, p, L, B < 1$

(5) $g > f$

(6) $L > B$

(7) $V_t > R_t$.

We found that this system of equations will in fact reproduce the growth-decay pattern of litigation which is empirically observed in several areas of jurisprudence. This module explores the analytic properties of the model.

2. EQUILIBRIA

While we now know that the growth-decay pattern occurs, we can be considerably more precise about the dynamic properties of the process. One such dynamic

characteristic which we wish to examine is the equilibrium point for the observed level of litigation, X^*.

Begin by setting the dynamic equations for ΔV_t and ΔR_t to zero and solving for the equilibria V^* and R^* respectively:

(8) $0 = g(L-V^*)$

(9) $0 = f(B-R^*)$.

Thus from (8)

(10) $V^* = L$

and from (9)

(11) $R^* = B$.

Then X_t is in equilibrium when both V_t and R_t are in equilibrium. That is

(12) $X^* = p(V^*-R^*)$,

or substituting (10) and (11) into (12),

(13) $X^* = p(L-B)$.

Once the period of doctrinal development is substantially over, then, the frequency of litigation for that particular class of cases will tend toward the quantity

(14) $X^* = p(L-B)$.

This level, therefore, is the residual uncertainty which is inherent in the law (L-B), filtered by the Court's responsiveness to this uncertainty (at rate p). The reader is encouraged to return to the first module and to re-examine the geometry of the problem in the light of this new information.

3. ELIMINATING THE UNOBSERVABLE

Since the states V_t and R_t are unobservable theoretical constructs, it would be convenient from an empirical perspective to be able to predict the change in X_t (which is observable) from a knowledge of previous values for X_t.

To do this we need to move from the expression of X_t as a function of V_t, R_t and the parameter p to some recursive function of X_t which eliminates the two unobservable states.

This, fortunately, can be accomplished by a standard mathematical technique. We will proceed by treating Δ as a *linear operator*, writing the system in matrix form, and then solving the linear system by Cramer's rule. We begin by briefly introducing the idea of linear operators.

Linear Operators

Operators are essentially instructions or rules which direct one to perform specific operations on functions.[1] Critical to our development is the distinction here between a class of operators, O, and the class of functions, Y. The operators which are introduced below are *linear*, in that they simultaneously satisfy two principles: homogeneity and additivity. Homogeneity requires that for operator O, function y and arbitrary constant c,

(15) $\quad O(cy) = c(Oy).$

Additivity requires that for arbitrary functions y_1 and y_2 and operator O,

(16) $\quad O(y_1+y_2) = Oy_1+Oy_2.$

One linear operator which we have utilized already in this module, without denoting it as such, is the *differencing operator*, Δ. Formally, let a function y be given and let h be any constant for which t+h is in the domain of y whenever t is. Then Δy, the first difference of y, is given by:

(17) $\quad \Delta y(t) = y(t+h) - y(t).$

[1] The reader may find any of the following sources useful in further understanding linear operators and their use: Samuel Goldberg, Introduction to Difference Equations (New York: Wiley, 1963); R.G.D. Allen, Mathematical Economics, 2nd edition (New York: St. Martin's, 1963); O. Lange, Introduction to Economic Cybernetics (New York: Pergamon, 1970); and Cortes et al., Systems Analysis for Social Scientists (New York: Wiley, 1974).

The number h is usually a positive integer and is called the differencing interval. In this module and in most applications, the differencing interval is taken to be unity. Hence, we normally define

(18) $\quad \Delta y(t) = y(t+1) - y(t).$

Applying the differencing operator twice gives

(19) $\quad \Delta(\Delta y) = \Delta^2 y,$

which is termed a second difference of y. The value of the function $\Delta^2 y$ at t (denoted $\Delta^2 y(t)$ or $\Delta^2 y_t$) is given by

(20) $\quad \Delta^2 y(t) = \Delta y(t+h) - \Delta y(t);$

and in general, the nth difference of the function y, Δ^n, is

(21) $\quad \Delta^n y = \Delta(\Delta^{n-1} y) \qquad n = 2, 3, 4, \ldots.$

Two additional linear operators which will be useful here are the *advancement operator*, denoted as E, and the *identity operator*, I. The advancement operator is defined as follows:

> Let a function y be given, and let h be any constant differencing interval. Then Ey is that function whose value at t, denoted as Ey(t) or Ey_t is given by

(22) $\quad Ey(t) = y(t+h).$

Again, conventional procedure is to set h = 1.

By applying the advancement operator twice, we have

(23) $\quad E[Ey(t)] = E[y(t+h)]$

(24) $\quad = y(t+2h).$

Notice then that when t denotes time, we may advance the time subscript on a state by applying the operator E. Each time E is applied, the time subscript is advanced by amount h (where h usually equals unity). Thus in general, if n is any real number,

(25) $E^n y(t) = y(t+nh)$.

Finally, the *identity operator*, I, is simply that operator which, when applied to any function y, produces a new function Iy which is identical to y. That is, for any t in the domain of y,

(26) $Iy(t) = y(t)$

(27) $I^n y(t) = I[y(t)] = y(t)$

and in general

(28) $I^n y(t) = y(t)$.

Notice that we may rewrite our definition for the differencing operator Δ in terms of the operators E and I. Specifically, since

(29) $\Delta y(t) = y(t+h) - y(t)$

we may equivalently write

(30) $\Delta y(t) = Ey(t) - Iy(t)$

or

(31) $\Delta = E-I$.

Intuitively, we are asserting that the same result is obtained by applying the operator Δ to any function as is obtained by applying the operator (E-I).

At a more general level, any two operators O_1 and O_2 are said to be *equivalent* if for any function y the functions $O_1 y$ and $O_2 y$ are equal. Thus there is an analogy between equivalence relations among operators and algebraic relations among real numbers. While proof is beyond the scope of this module, *we can manipulate the linear operators Δ, E and I in equivalence equations just as we do algebraic quantities in numerical equations.* This is an extremely powerful result, one which we will underscore through several examples.

If we begin, for example, with a numerical equation

(32) $Y_t = AY_{t-1} + B$,

where A and B are arbitrary constants, we may apply the operator E to both sides of (32) to obtain

(33) $EY_t = EAY_{t-1} + EB$

or

(34) $Y_{t+1} = AY_t + B$.

Note that the advancement operator when applied to a constant simply returns the constant (EB = B). Operating on (32) by E^2 produces

(35) $E^2 Y_t = E^2 AY_{t-1} + E^2 B$

(36) $E(EY_t) = E(EAY_{t-1}) + E(EB)$

(37) $E(Y_{t+1}) = E(AY_t) + EB$

(38) $Y_{t+2} = AY_{t+1} + B$.

On the other hand, had we applied the differencing operator Δ to both sides of (32) we would obtain

(39) $\Delta Y_t = \Delta AY_{t-1} + \Delta B$,

and from the definition of Δ, this produces

(40) $Y_{t+1} - Y_t = AY_t - AY_{t-1} + 0$

(41) $Y_{t+1} = (1+A)Y_t - AY_{t-1}$.

Notice that the operator Δ when applied to a constant gives zero ($\Delta B = EB - IB = B - B = 0$).

The equivalency of Δ and (E-I) may be seen if we substitute (E-I) for Δ in Equation (39):

(42) $(E-I)Y_t = (E-I)AY_{t-1} + (E-I)B$

(43) $EY_t - IY_t = EAY_{t-1} - IAY_{t-1} + EB - IB$

or

(44) $Y_{t+1} - Y_t = AY_t - AY_{t-1} + B - B$

(45) $Y_{t+1} = (1+A)Y_t - AY_{t+1}$,

which is identical to Equation (41).

The equivalency of Δ with (E-I) is particularly useful in simplifying both numerical and functional equations. For example, if we have

(46) $Y_{t+1} = \Delta^2 Y_t$,

we may write

(47) $Y_{t+1} = (E-I)^2 Y_t$

(48) $Y_{t+1} = (E^2-2EI+I^2)Y_t$

(49) $Y_{t+1} = E^2Y_t - 2EIY_t + I^2Y_t$

(50) $Y_{t+1} = Y_{t+2} - 2Y_{t+1} + Y_t$

or

(51) $Y_{t+2} - 3Y_{t+1} + Y_t = 0.$

In general, then, the linear operators Δ, E and I obey the same algebraic laws as ordinary constants. As we shall see, this is quite powerful in treating our problem of Supreme Court decision-making. Before attempting that section, however, the reader should be able to work the following problems.

Question 1:

a. $\Delta^3 = ?$ in terms of E and I?

b. Simplify the expression $\Delta\Delta EY_t$.

c. If $Y_{t+1} = Y_t + B$, what is the result of applying the operator Δ^2 to both sides of the equations?

d. Moving from numerical to functional equations, if $y(x) = x^3$ (taking h = 1) then for example

$$\Delta y(2) = y(x+1) - y(x)$$
$$= y(2+1) - y(1)$$
$$= y(3) - y(2)$$
$$= 3^3 - 2^3$$
$$= 27 - 8$$
$$= 19.$$

In general, then, find $\Delta y(t)$ if $y(t) = t^2$ and h = 1.

e. Find $\Delta y(t)$ if $y(t)$ is given by

$$y(t) = 4 - 2t + t^2$$
$$y(t) = 6t(1-t)$$
$$y(t) = t/2.$$

Do not assume h = 1.

Application of Linear Operators: Solution of the System by Cramer's Rule

The technique presented here is a general strategy which is applicable when the underlying dynamic equations of a system are linear. The goal, again, is to obtain a recursive solution for the observable state, X_t, and thereby eliminate the unobservable states V_t and R_t. We express the system in matrix form, then solve using Cramer's rule.[2]

In order to rewrite Equations (1) and (3) in matrix form, use Δ as a linear operator and obtain

(52) $(\Delta+g)V_t = gL$

(53) $(\Delta+f)R_t = fB$

(54) $X_t - pV_t + pR_t = 0.$

We have simply used Δ as a constant and collected terms on V_t and R_t. The rewrite Equations (52), (53) and (54) as:

---

[2]For this elementary dynamic system a simpler derivation is possible. Solution by Cramer's rule is a more general strategy, particularly useful if the dynamic system is more complex. It is thus presented here.

$$(55)\qquad \begin{bmatrix} (\Delta+g) & 0 & 0 \\ 0 & (\Delta+f) & 0 \\ -p & p & 1 \end{bmatrix} \begin{bmatrix} V_t \\ R_t \\ X_t \end{bmatrix} = \begin{bmatrix} gL \\ fB \\ 0 \end{bmatrix}.$$

We wish then to solve for X_t.

Question 2: Write the following linear system in matrix form.

$$X_t = Y_t + (1-g)Z_t$$

$$Z_t = AX_t + BY_t$$

$$Y_t = -CX_t - DZ_t + F.$$

Applying Cramer's rule.[3]

$$(56)\qquad X_t = \frac{\begin{vmatrix} (\Delta+g) & 0 & gL \\ 0 & (\Delta+f) & fB \\ -p & p & 0 \end{vmatrix}}{\begin{vmatrix} (\Delta+g) & 0 & 0 \\ 0 & (\Delta+f) & 0 \\ -p & p & 1 \end{vmatrix}}.$$

The numerator is given by (expanding around the 3rd row):

$$(57)\qquad -p(-1)^{3+1}\begin{vmatrix} 0 & gL \\ (\Delta+f) & fB \end{vmatrix} + p(-1)^{3+2}\begin{vmatrix} (\Delta+g) & gL \\ 0 & fB \end{vmatrix} + 0$$

---

[3]Cramer's rule states that:

If $A = (a_{ij})$ is nonsingular $n \times n$ matrix, then the linear system $AX = \beta$ has a unique solution $X = (x_k)$ with

$$x_k = \frac{\det A^{(k)}}{\det A}$$

where $A^{(k)}$ denotes the matrix formed by replacing the kth column of A with the n-tuple $\beta = (b_k)$.

$$(58)\qquad = -p(+1)[0-gL(\Delta+f)] + p(-1)[fB(\Delta+g) - 0]$$

$$(59)\qquad = pgL(\Delta+f) - pfB(\Delta+g).$$

Then the denominator of (56), expanding around the third column is:

$$(60)\qquad 0 + 0 + 1(-1)^{3+3}\begin{vmatrix} (\Delta+g) & 0 \\ 0 & (\Delta+f) \end{vmatrix}$$

$$(61)\qquad = (\Delta+g)(\Delta+f).$$

Therefore,

$$(62)\qquad X_t = \frac{pgL(\Delta+f) - pfB(\Delta+g)}{(\Delta+g)(\Delta+f)}$$

or cross-multiplying

$$(63)\qquad \Delta^2X_t + (f+g)\Delta X_t + fgX_t = pgL\Delta + pgLf + pfB\Delta - pfBg.$$

Recall, however, that Δ applied to a constant = 0, so (63) may be rewritten as

$$(64)\qquad \Delta^2X_t + (f+g)\Delta X_t + fgX_t = pgfL - pgfB$$

or

$$(65)\qquad \Delta^2X_t + (f+g)\Delta X_t + fgX_t = pgf(L-B).$$

To eliminate the Δ's from the left-hand side of (65), utilize the equivalency

$$(66)\qquad \Delta = E-I$$

and substitute this into (65):

$$(67)\qquad (E-I)^2X_t + (f+g)(E-I)X_t + fgX_t = pgf(L-B).$$

Expanding and collecting terms gives

$$(68)\qquad E^2X_t + (f+g-2)EX_t + (1-f-g+fg)X_t = pgf(L-B).$$

Since

$$(69)\qquad EX_t = X_{t+1}$$

and

(70) $$E^2X_t = X_{t+2},$$

we may finally write (68) as

(71) $$X_{t+2} + (f+g-2)X_{t-1} + (1-f-g+fg)X_t = pgf(L-B).$$

We thus have an expression from which we may deduce change in the observed frequency of litigation, X_t, without a knowledge of the unobservable states of the system. The empirical usefulness of this, and similar models may thus be greatly enhanced by the technique.

It is important to notice in addition that the system is in fact 2nd order. Thus, to predict a future level of litigation (at t+1), we must know both the current level of litigation (at t) and the past level (at t-1). This implies that the Court responds not only to its present behavior but has a memory of its past activities which also impinges on its decision mak. .

Question 3: Given

$$\Delta X_t = AX_t + BY_t$$

$$\Delta Y_t = CX_t - DY_t + F$$

Solve the system for Y_t by writing in matrix form and using Cramer's rule. A system of two first-order equations is equivalent to one equation of what order?

---

4. CONCLUSION

This module began by suggesting that discretionary review by the Supreme Court exhibits a systematic pattern of change over time. We have argued that this dynamic pattern is related to the process of doctrinal development by the Court.

It was suggested that a major shift in interpretation signals a period of increased uncertainty about the Court's behavior, along with increased demands by legal participants for access to the Court. The Court's caseload in this area of litigation thus exhibits a period of growth as the Court attempts to develop a new set of rules—legal norms. Once a new doctrine is articulated, the level of uncertainty about the Court will diminish, and with it the demand for access. Litigation will return to more routine levels.

The dynamic formulation presented here will, in fact generate the characteristic features of the observed time-paths. The puzzling phenomenon—a particular pattern of growth and then decay to a limit—was thus shown to be a consequence of the deductive logic of the model under a particular substantive interpretation.

In addition to reproducing the observed phenomenon, the deductive structure of the model makes it possible to gain several insights about the process of discretionary review. It is worth pointing out, however, that the formalization is useful only in the analysis of synchronic change: dynamics which result from a constant structure. We can predict a history of discretionary review by the Supreme Court once a doctrinal shift occurs; we cannot, however, predict from the model when or how the shift will in fact take place.

Probably one must look to the cross-sectional analyses of judicial gatekeeping. These studies are worth noting since their logic is so different from the approach taken here. By asking "what variables are related to other variables" during some frozen instant in time, these studies consider more directly the effects of population densities, industrialization, socioeconomic factors, the structure of appellate courts below, and numerous other variables.

Certainly this is a useful and necessary approach to the problem of access to the Supreme Court. But a major

conclusion here is that while cross-sectional analyses teach important lessons, they do not tell the whole story: during periods of doctrinal development there is a time-dependent component to discretionary review which is better treated with an explicitly dynamic formulation.

5. ANSWERS TO QUESTIONS

1a. $\Delta^3 = (E-I)^3$

$= (E-I)(E-I)^2$

$= (E-I)(E^2-2EI+I^2)$

$= E^3 - 3E^2 + 3E - I.$

b. $\Delta\Delta EY_t = (E-I)(E-I)(E)Y_t$

$= (E^2-2EI+I^2)EY_t$

$= (E^3-2E^2I+EI^2)Y_t$

$= Y_{t+3} - 2Y_{t+2} + Y_{t+1}.$

c. $\Delta^2Y_{t+1} = \Delta^2Y_t + \Delta^2B$

$(E^2-2EI+I^2)Y_{t+1} = (E^2-2EI+I^2)Y_t + (E^2-2EI+I^2)B$

$Y_{t+3} - 2Y_{t+2} + Y_{t+1} = Y_{t+2} - 2Y_{t+1} + Y_t + B - 2B + B$

$Y_{t+3} - 3Y_{t+2} + 3Y_{t+1} - Y_t = 0.$

d. $\Delta y(t) = y(t+1) - y(t)$

$= (t+1)^2 - t^2$

$= t^2 + 2t + 1 - t^2$

$= 2t - 1.$

e. $y(t) = 4 - 2t + t^2$

$\Delta y(t) = 4 - 2(t+h) + (t+h)^2 - 4 + 2t - t^2$

$= 4 - 2t - 2h + t^2 + 2th + h^2 - 4 + 2t - t^2$

$= h^2 + 2th - 2h$

$y(t) = 6t(1-t)$

$\Delta y(t) = 6(t+h)(1-t+h) - 6t(1-t)$

$= 6h(1-h).$

$y(t) = t/2$

$\Delta y(t) = (t+h)/2 - t/2.$

2. $\begin{bmatrix} \Delta & \Delta & -(1-g) \\ -A & -B & \Delta \\ +C & \Delta & D \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ E \end{bmatrix}.$

A linear system of two first-order equations is equivalent to one second-order equation.

3. In matrix form:

$$\begin{bmatrix} (\Delta-A) & -B \\ -C & (\Delta+D) \end{bmatrix} \begin{bmatrix} X_t \\ Y_t \end{bmatrix} = \begin{bmatrix} 0 \\ F \end{bmatrix}.$$

By Cramer's rule

$$Y_t = \frac{\begin{vmatrix} (\Delta-A) & 0 \\ -C & F \end{vmatrix}}{\begin{vmatrix} (\Delta-A) & -B \\ -C & (\Delta+D) \end{vmatrix}} = \frac{F\Delta-FA}{(\Delta+D)(\Delta-A)-CB} = \frac{F\Delta-FA}{\Delta^2+(D-A)\Delta-(AD+CB)}$$

$[\Delta^2 + (A-D)\Delta - (AD+CB)]Y_t = FA$

$[E^2 - 2EI + U^2 + (A-D)E + (A-D)I - (AD+CB)]Y_t = FA$

$Y_{t+2} + (A-D-2)Y_{t+1} + (I+A-D-AD-CB)Y_t = FA.$

6. BIBLIOGRAPHY

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STUDENT FORM 1

Request for Help

Return to:
EDC/UMAP
55 Chapel St.
Newton, MA 02160

Student: If you have trouble with a specific part of this unit, please fill out this form and take it to your instructor for assistance. The information you give will help the author to revise the unit.

Your Name_____ Unit No._____

Page_____
- ○ Upper
- ○ Middle
- ○ Lower

OR

Section_____

Paragraph_____

OR

Model Exam
Problem No._____

Text
Problem No._____

Description of Difficulty: (Please be specific)

Instructor: Please indicate your resolution of the difficulty in this box.

○ Corrected errors in materials. List corrections here:

○ Gave student better explanation, example, or procedure than in unit. Give brief outline of your addition here:

○ Assisted student in acquiring general learning and problem-solving skills (not using examples from this unit.)

Instructor's Signature_____

Please use reverse if necessary.

STUDENT FORM 2

Unit Questionnaire

Return to:
EDC/UMAP
55 Chapel St.
Newton, MA 02160

Name_____ Unit No._____ Date_____

Institution_____ Course No._____

Check the choice for each question that comes closest to your personal opinion.

1. How useful was the amount of detail in the unit?

____Not enough detail to understand the unit
____Unit would have been clearer with more detail
____Appropriate amount of detail
____Unit was occasionally too detailed, but this was not distracting
____Too much detail; I was often distracted

2. How helpful were the problem answers?

____Sample solutions were too brief; I could not do the intermediate steps
____Sufficient information was given to solve the problems
____Sample solutions were too detailed; I didn't need them

3. Except for fulfilling the prerequisites, how much did you use other sources (for example, instructor, friends, or other books) in order to understand the unit?

____A Lot ____Somewhat ____A Little ____Not at all

4. How long was this unit in comparison to the amount of time you generally spend on a lesson (lecture and homework assignment) in a typical math or science course?

Much	Somewhat	About	Somewhat	Much
____Longer	____Longer	____the Same	____Shorter	____Shorter

5. Were any of the following parts of the unit confusing or distracting? (Check as many as apply.)

____Prerequisites
____Statement of skills and concepts (objectives)
____Paragraph headings
____Examples
____Special Assistance Supplement (if present)
____Other, please explain_____

6. Were any of the following parts of the unit particularly helpful? (Check as many as apply.)

____Prerequisites
____Statement of skills and concepts (objectives)
____Examples
____Problems
____Paragraph headings
____Table of Contents
____Special Assistance Supplement (if present)
____Other, please explain_____

Please describe anything in the unit that you did not particularly like.

Please describe anything that you found particularly helpful. (Please use the back of this sheet if you need more space.)